A RME-based Trigonometry Design Research for Vocational High Schools Students

Noveria Ningsih, Edwin Musdi, Ali Asmar, I Made Arnawa

Abstract— This study aimed to produce a trigonometric learning path based on RME to help students find phytagorean concepts, trigonometric comparisons, trigonometric comparison values, inverse trigonometric comparisons, trigonometry special angles and trigonometry related angles. This research was a Gravemeijer and Cobb design study which consists of three stages, namely the experimental preparation stage, the experimental design stage, and the retrospective analysis stage. In the first stage, a Hypothetical Learning Trajectory (HLT) design and student worksheets used the RME approach through literature studies. In the second stage, the HLT trial was applied on 6 students in small groups with high, medium, and low abilities. In the third stage, an analysis of the test results was applied so that a learning flow is produced. Overall, the RME-based trigonometric topic learning design can be used for learning. The contextual problems given are close to students' daily lives so that they can stimulate students to find trigonometric concepts. Thus, it can be concluded that HLT on the topic of RME-based trigonometry can facilitate students in understanding and finding trigonometric concepts.

Index Terms— Instructional Design, Trigonometry, RME, Learning Trajectory, Design Research.

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1 Introduction

TRIGONOMETRY is one of the mathematics subjects in Vocational High Schools which has an important role, because it is a mathematical material that we often encounter in everyday life, either directly or indirectly. Astrology and building construction are greatly assisted by the presence of trigonometry [1]. Trigonometry comes from the Greek language, namely "tri", "gonios" and "metros", each of which means three, line, and measure. In accordance with this lexical meaning, the trigonometric ratio of an angle is obtained from the ratio of the dimensions of the three lines corresponding to that angle. Thus there are prerequisites that must be met before discussing trigonometry. The prerequisite is material on projections, angles, and the congruence nature of triangles [2].

Indonesia's achievements in mathematics are still not satisfactory, including trigonometry. Based on the results of a survey by the TIMSS institute in 2015, Indonesia was ranked 45th out of 50 countries with a score of 397. The institution tests basic competencies which include numbers, algebra, geometry, trigonometry and measurement. The same thing is seen in the achievements obtained during the PISA test, in which Indonesia is ranked 65th with 72 countries participating in the same year [3]. Therefore trigonometry is an interesting study to research. In addition there are several problems related to learning trigonometry, including trigonometry is considered difficult and feared by students, and students memorize formulas and do not understand concepts [4].

Based on the phenomena that have been described, the authors argue that a learning design is needed to overcome these problems. This learning design was developed using Realistic Mathematics Education (RME). The RME approach was chosen in developing a trigonometric learning path because several studies have shown that LIT is very powerful for building students' understanding of mathematical concepts [5], [6], [7], [8], [9], [10]. The learning flow is designed at the beginning of the lesson, students are given contextual problems that can be solved using their informal knowledge. Contextual problems will also facilitate students to use their own symbols or their own strategies. This process is called horizontal mathematics. After experiencing a similar process and being empowered by simplification and formalization [11], students will use more formal language or strategies in solving contextual problems. The journey, which will lead students to rediscover formal mathematics, is called vertical mathematics [12, 13, 14]. Based on the description above, the aim of this study is to design HLT on the topic of trigonometry using the RME approach. HLT is implemented in grade X of Vocational High Schools students. The topic of trigonometry consists of the phytagorean theorem, trigonometric comparisons, trigonometric ratio values, inverse trigonometric comparisons, trigonometry special angles, and trigonometry related angles.

2 Method

The method in this research was design research by Gravemeijer and Cobb. The design research was conducted by designing learning trigonometric topics using the RME approach. Implementation of this research is assisted using instruments in the form of Hypothetical Learning Trajectory (HLT) and student worksheets. This design research consists of three stages, namely, the preparation of the experiment, the design of the experiment (The Design Experiment), and retrospective analysis (The Retrospective Analysis) [15].

In the preparation stage of the experiment (preparing the experiment) there were several literature studies on the topic to be used to design the HLT. This stage aims to collect all the data and materials needed to design HLT. The next activity is the HLT design. HLT is dynamic in nature so it can be revised depending on the trial process.

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At the experimental design stage (The Design Experiment) the HLT design was tested in a small group consisting of six grade X students of SMKN 1 Pantai Cermin. At this stage, the writer acts as a researcher and teacher to collect data. Six students were divided into 2 groups, each group consisting of 3 students with high, medium and low abilities. The skills of the students selected vary from high to low. The teacher assists in selecting students.

In the retrospective analysis stage (The Retrospective Analysis) the results of data analysis from the teaching experiment stage are used to develop designs for further learning. Data collection techniques in this study were field notes, video recordings of learning, observation, and interviews. Data were collected to describe the implementation of HLT. This design research scheme is illustrated in the following figure.

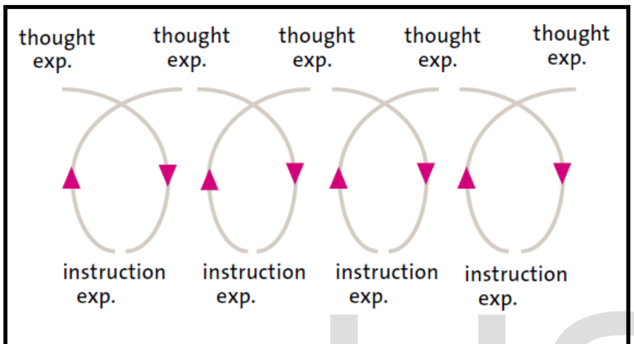

Figure 1.Design Research Cycle [15]

3 RESULTS AND DISCUSSION

The description of the RME-based trigonometric topic design research for SMK students from the experimental preparation stage to the retropective analysis is described as follows.

3.1 PREPARING FOR THE EXPERIMENT

This stage begins with a review of the literature in the form of a mathematics class X student book in the 2013 curriculum and several research journals. Based on the literature review, the HLT was designed with the objectives to be achieved.

TABLE 1
THE DESIGN OF HLT

Activities on HLT Designed
1. The concept of the Pythagorean theorem on a right triangle. 1.1 Finds the concept of the Pythagorean theorem on a right triangle through the context of calculating the height of a wall.
2. The concept of trigonometric comparisons in a right triangle. 2.1 Find the concept of trigonometric comparisons in a right triangle using the concept of similarity
3. The concept of the trigonometric ratio of a right triangle 3.1 Finds the trigonometric ratio value of a right triangle through the context of measuring the height of the flagpole. 3.2 Using the trigonometric ratio value of a right triangle to solve more complex problems
4.The concept of inverse trigonometric ratios. 4.1 Find the concept of inverse trigonometric ratios in a right triangle using the concept of similarity 4.2 Using the inverse trigonometric ratio of a right triangle to solve more complex problems
5. Concept of special angle trigonometric value. 5.1 Find the concept of special angle trigonometric values using the comparison of the right triangle and the Pythagorean theorem. 5.2 Uses the concept of preferential angle values to solve more complex problems.
6. The concept of trigonometric comparison of related angles. 6.1 Find the concept of trigonometric comparison of related angles in each quadrant using the context of the ship's voyage.

3.2 THE DESIGN EXPERIMENT

HLT is implemented for six students with various abilities. The students were divided into 2 groups, each consisting of three students with high, medium, and low abilities. The following describes the results of the small group evaluation.

1. Finding the Concept of the Pythagorean Theorem in a right triangle.

The purpose of activity 1.1 is for students to find the concept of the Pythagorean theorem through the context of calculating the height of a wall. This problem will stimulate students to think about how to find the height of the wall. Through this activity, it is hoped that the process of calculating the height of the wall will be determined by finding the area of a large square first. So that from this process students can find the concept of the Pythagorean theorem. Furthermore, students are expected to understand that the Pythagorean theorem can be used by students to determine one of the unknown sides of a right triangle.

Group 1 can directly answer and calculate the height of the wall with the Pythagorean theorem, while in group 2 the sketched right triangle of the contextual problem is drawn in a square so that the height of the wall can be found from the calculation of the area of the square. However, group 2 could not find the concept of the Pythagorean Theorem from the process of finding the height of the wall. To anticipate this, the teacher gave a probing question to group 2, so that students could find the Pythagorean concept.

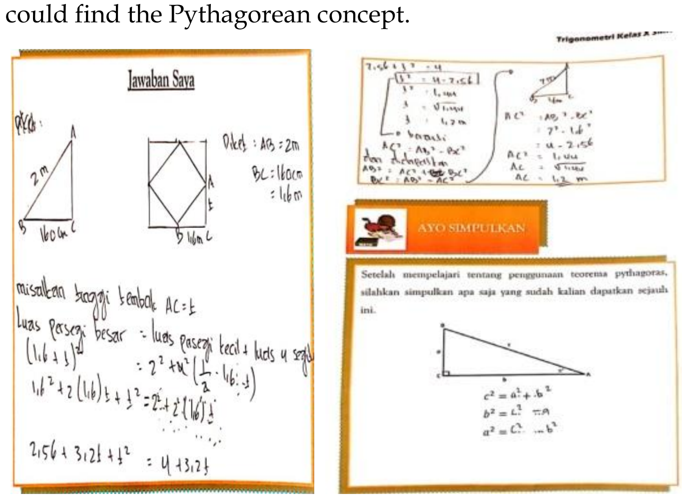

Figure 2.Students' answer sheet in Activity 1.1

The probing question given is to provide directions for students to find the Pythagorean theorem from the process of calculating the height of the wall using a square area. From the elaboration of the area of the square, students can find that the square of the hypotenuse is the sum of the squares of the base and height of a right triangle, the square of the base of the triangle is the square of the hypotenuse minus the square of the height of the triangle, likewise the square of the height of the triangle is the square of the hypotenuse minus the square of the base of the triangle. Retrospective analysis of this problem, namely this problem has been able to stimulate students to construct the concept of using the Pythagorean Theorem on a right triangle.

2. Finding the concept of a right-angled triangle trigonometric comparison

The purpose of activity 2.1 is for students to find the concept of trigonometric comparisons in a right triangle using the concept of similarity triangles. This activity is carried out by providing the opportunity for students to find comparisons to similar triangles through contextual problems that arise in the activity. From the comparisons of similar triangles students can deduce the concept of trigonometric comparisons. In this activity, prediction and anticipation are able to direct students in concept discovery.

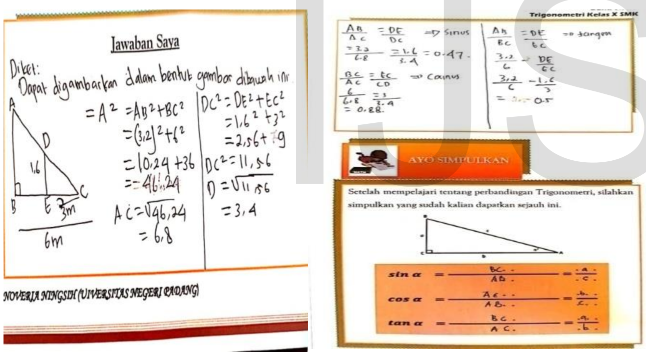

Figure 3 .Students' answer sheet in Activity 2.1

Based on Figure 3, it can be seen that, in this activity, it can be seen that the two groups were able to find the concept of trigonometric comparisons using the congruent triangle concept of the given contextual problems. Activity 2.1 can stimulate students to understand the concept of trigonometric comparisons. After students complete activity 2.1 as shown above, the teacher asks students to write down conclusions about trigonometric comparisons. Then the teacher asks the students "If you change the position of the angle ∝ and you rotate the rectangular triangle or change its position, can you still determine the trigonometric ratio". After the students heard and thought, they finally found that the trigonometric ratio for sine, cosine and tangent. Then one of the students from group 2 said "So if you already know the concept of trigonometric comparisons, like any right triangle image, the trigonometric ratio can be determined".

3. Find the trigonometric ratio value in a right triangle

The purpose of activity 3.1 is so that students can determine the value of the trigonometric ratio in a right triangle. Trigonometric comparison values can be found by students if students have mastered the concept of trigonometric comparisons at the previous meeting. This activity is carried out by providing opportunities for students to explore the problem of measuring the flagpole.

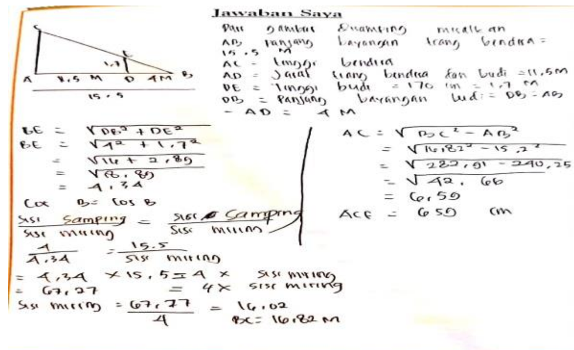

Figure 4.Students' answer sheet in Activity 3.1

In solving the problem in activity 3.1, the two groups already understood the contents of the problem and were able to solve it in their own way according to the concept of trigonometric comparisons in the right triangle they had obtained in activity 2.1. This is consistent with the characteristics of RME, namely intertwining, related to other topics. Each group describes the problem in the form of a right triangle. From the description of the problem, the students found two right triangles. The height of the flagpole is found by the students using one of the trigonometric comparisons of the two right triangles.

The purpose of activity 3.2 is so that students can use the trigonometric ratio value in a right triangle to solve more complex problems. This activity is carried out by providing opportunities for students to explore the problem of measuring the height of the flagpole using the elevation angle shown by the clinometers.

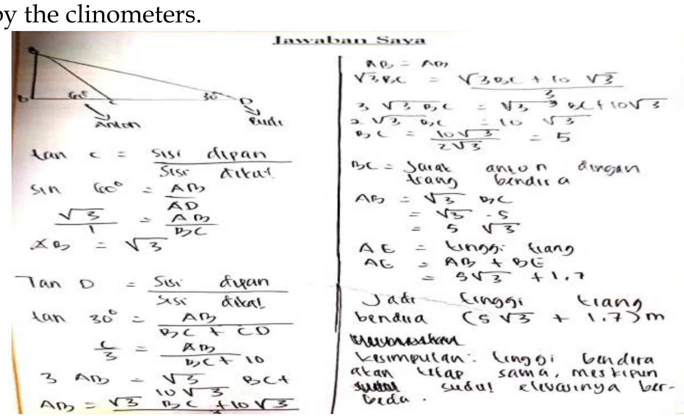

Figure 5.Students' answer sheet in Activity 3.2

At the time of carrying out this activity 3.2, predictions and anticipations that occur are able to lead students to be able to use the trigonometric comparison value of a right triangle to find the height of the flagpole and be able to deduce the relationship of an object with its elevation point of view.

4. Finding the concept of inverse trigonometric ratios in a

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right triangle

The purpose of activity 4.1 is for students to find the concept of inverse trigonometric comparisons in right triangles. This activity is carried out by providing the opportunity for students to find inverse trigonometric comparisons through contextual problems measuring images by a child.

The activity was started with observing three different pictures. Each product has different bruto, netto and tarra listed in the package. Following the procedure, students discussed in group trying to assume the definition of bruto, netto and tara and finding how they are related. Both groups have completed this contextual problem successfully.

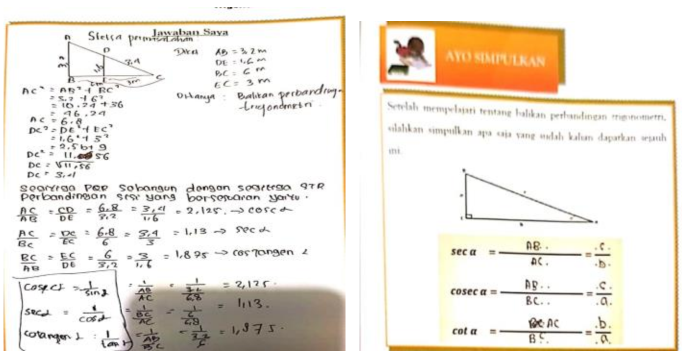

Figure 6.Students' answer sheet in Activity 4.1

In this activity, predictions and anticipations are designed to go as expected. The teacher's anticipations are able to direct students in finding the concept of inverse trigonometric comparisons. It can be seen that the two groups are able to find the concept of inverse trigonometric comparisons by using the congruent triangle concept from the given contextual problems. After students complete activity 4.1 as shown above, the teacher asks students to write conclusions about inverse trigonometric comparisons contained in student worksheets.

The purpose of activity 4.2 is so that students can use the trigonometric ratio inverse value in a right triangle to solve more complex problems. This activity is carried out by giving students the opportunity to determine the eagle's height from the ground.

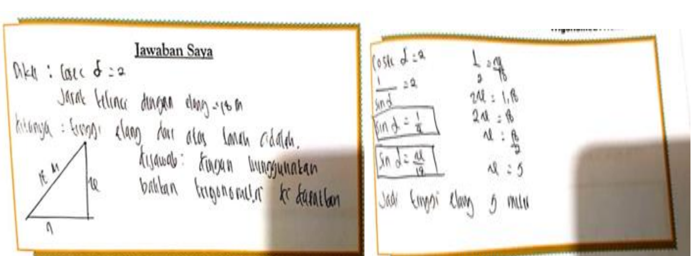

Figure 7.Students' answer sheet in Activity 4.2

In solving problems in activity 4.2, the two groups already understood the content of the problem and were able to solve it with the concepts they had obtained in activity 4.1. The contextual problem given is described by the students in the form of a right triangle. The students found the eagle's height from the ground using one of the trigonometric inverse ratios concepts. Overall students can understand the problems contained in this activity. So that with this problem students can determine the value of the trigonometric ratio inverse and use it in solving various related problems.

5. Finding the Concept of Special Angle Trig Value

Activity 5.1 aims to enable students to find the concept of a special angle trigonometric value. This activity is carried out by giving students the opportunity to use the concept of the angle comparison in a right triangle. The anticipation given by the teacher is able to stimulate students to find special angle values. The special angle found in contextual problems is taken as angles 60^{o} .

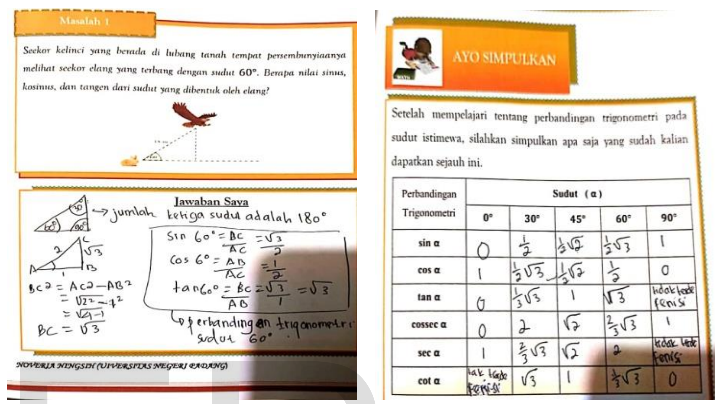

Figure 8.Students' answer sheet in Activity 5.1

The students' answers above illustrate that contextual problems can bridge students to find concepts. In general, the two groups have done well, namely by determining the ratio of the angles of the right triangle and then determining the ratio of the unknown sides with the Pythagorean theorem. These students' findings have led students to find special angle values. In this activity the two groups only found the trigonometric ratio at special angles 30^{o}, 45^{o}, and 60^{o}. Then during the discussion the teacher asked the students "What about the special angles 0^{o} and 90^{o}, can this method also be used? ". After the students heard and thought, they finally suggested that these two special angles do not form a right triangle. So for angles 0^{o} and 90^{o} "this method cannot be used. Responding to the student's answer, the teacher emphasized that to determine the trigonometric ratio of the angles 0^{o} and 90^{o} it is more appropriate to calculate using the point coordinate method.

This activity 5.2 aims to enable students to use the concept of special angle values in more complex problems. This activity is carried out by providing opportunities for students to understand the context for determining the height of the flagpole as seen by two teachers with different elevation angles.

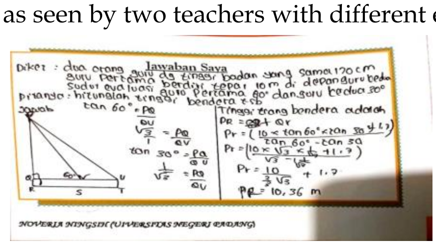

Figure 9.Students' answer sheet in Activity 5.2

In solving problems in activity 5.2, both groups understood the content of the problem and were able to solve it with the concepts they had obtained in activity 5.1. The contextual problem given is described by the students in the form of a right triangle. The elevation angle contained in the problem is a special angle studied in activity 5.1

6. Finding the Concept of Trigonometric Comparison Value of Correlated Angles

The purpose of this activity is so that students can find the corresponding angle values in each quadrant. This activity is carried out by providing opportunities for students to explore contextual problems related to related angles. The anticipations made by the teacher can stimulate students in concept discovery. In activity 6.1, the contextual problem used is the context of the ship's shipping direction.

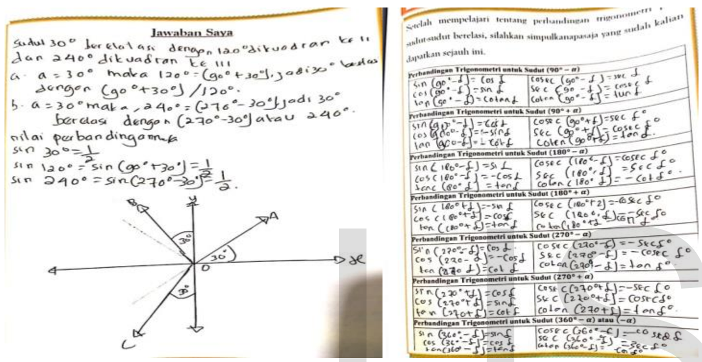

Figure 10.Students' answer sheet in Activity 6.1

Based on the picture above, it can be seen that the students have understood the related angle and have been able to deduce the trigonometric ratio of the related angles in each quadrant.

3.3 RETROSPECTIVE ANALYSIS

After the learning process ends, researchers and teachers review the extent of the implementation of the learning design. There are students who are able to immediately solve problems without using help from the teacher. Some students seem to understand very quickly and are able to solve every problem and can find concepts on their own. But there are also students who still have difficulty finding concepts on their own.

Based on the results of the design trial, a learning trajectory was obtained that began with contextual problems, understanding the problem, creating a solution model and concluding. Activities that are designed to produce various kinds of student answers in which each student's answer is anticipated in order to achieve learning objectives. This trial process uses the RME approach which contains 4 principles, namely guided reinvention, didactical phenomenology, emerging models, and horizontal and vertical mathematization [16].

The principle of guided reinvention is illustrated by the process of finding the concept of the 4 trigonometric learning objectives given. Finding concepts starting from informal procedures, then informal strategies are useful for moving towards more formal procedures. The principle of didactical phenomenology is illustrated by the use of the phenomena used in each activity. The principle of emerging models is illustrated in the answers of students who try to solve problems with their own models. The principle of horizontal and vertical mathematization is illustrated in the process of students solving problems to obtain conclusions and general concepts to achieve goals.

Overall, the RME-based trigonometric topic learning design can be used for learning. The contextual problems given are close to students' daily lives so that they can stimulate students to find trigonometric concepts. However, there are several things that need to be updated, such as the HLT 6 problem, the problem that is presented is only one, although in learning students can find the concept of trigonometric comparison of related angles, it is better if this related problem is added because this sub topic is difficult from other sub topics. This is intended so that students do not experience difficulties in further experiments.

4 CONCLUSION AND SUGESSTION

Based on the results and discussion that have been described, the authors can conclude that the RME-based trigonometric topic mathematics learning design is a learning design that can contribute to the development of students' thinking skills. Learning becomes more meaningful because it starts with contextual problems. Thus, it can be concluded that HLT on RME-based trigonometric topics can facilitate students in understanding and finding the concepts of trigonometric topics.

Therefore, it is hoped that the RME-based learning design for the trigonometric topic can be developed even better so that the learning objectives are achieved more optimally.

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